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THE EXPECTED TIME TO FIRST FAILURE

A. M. FREUDENTHAL

OHIO STATE UNIVERSITY RESEARCH FOUNDATION

TECHNICAL REPORT AFML-TR-66-37

FEBRUARY 1966

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THE EXPECTED TIME TO FIRST FAILURE

A. M. FREUDENTHAL

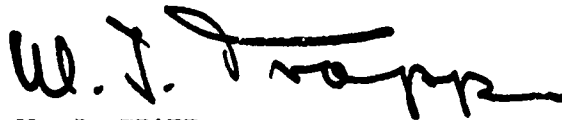*OHIO STATE UNIVERSITY RESEARCH FOUNDATION*

FOREWORD

This report was prepared by Dr. A. M. Freudenthal, New York, N. Y. under USAF Contract AF 33(657)-8741. This contract was initiated under Project No. 7351, Metallic Materials", Task No. 735106,"Behavior of Metals". The contract was administered by the Ohio State University Research Foundation. The work was monitored by the Metals and Ceramics Division, AF Materials Laboratory, Research and Technology Division, Air Force Systems Command, Wright-Patterson Air Force Base, Ohio, under the direction of Mr. W. J. Trapp.

This report covers the period of work, January 1, 1965 to June 30, 1965.

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ABSTRACT

A new approach to structural reliability analysis based on
order statistics is introduced by considering the expected time
to the first failure in a fleet of specified magnitude. Because
in the design of large structural units, such a transport air-
craft, failure of even a single unit must be prevented, reliabil-
ity analysis and design for a "mean time to failure" seems to be
an unjustified extension of the use of methods of reliability
analysis developed for inexpensive mass-produced items of rela-
tively short service lives to the reliability assessment of ex-
pensive, large units. A method for the estimate of the expected
time to the first failure is outlined and the implications of
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cussed.

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I. INTRODUCTION

The practical use of the mathematical theory of probability and its intuitive meaning in the analysis of structural reliability is based on its connection with real or conceptual experiments, such as the counting of exceedances of specified intensities of loads or the observation of the time-intervals between exceedances ("return periods"), the counting of the number of failures in mechanical tests or in operation of critical elements of mechanical systems, or the observation of intervals between failures. For the theory to be meaningful, the "statistical population" must be clearly defined by specifying the possible outcomes of the counts or observations. Such specification is of a physical rather than a probabilistic nature and thus determines the physical character of the probabilistic model. The "random variable" X is defined over the population in such a way that specific numerical values, either discrete or continuous, are assigned to each outcome. A function of x representing the probability of an outcome equal to or smaller than $X = x$ is the "probability function" $P(x)$, which is the probability of occurrence of outcomes $X \leq x$, while $R(x) = 1 - P(x)$ is the probability of outcomes $X > x$ in large (theoretically infinite) numbers of experiments or observations.

In a physical situation the probability function is usually unknown and has therefore to be determined either

(a) by statistical inference from a necessarily limited number n of outcomes ("sample" of size n), or

(b) by theoretical reasoning based on
(1) a conceptual experiment, or
(2) a physical or engineering concept.

In the first case statistically significant outcomes must be available and presented in a form suitable for inference of the population function $P(x)$ from the plotting position of the n sample points $F(x_m)$ where $m = 1, 2, \ldots n$.

In the second case the population function is directly derived from a probabilistic or from a physical model without reference to experiment or observation.

The limitation of the method of statistical inference in structural reliability analysis can be easily illustrated by considering the mean (cumulative) frequency $F(x_m)$ of the m-th obser-

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vation x_m of a continuous random variable X in a sample of size n, in which all observations have been arranged in increasing order

$$F(x_m) = m/(n + 1) \qquad (1)$$

This expression has been proposed[1] as the "plotting position" $F(x_m)$ of the m-th observation x_m on probability paper. Therefore, for a sample size $n = 10$ the range of (cumulative) frequencies is enclosed between $F(x_1) = 1/11 = 0.091$ and $F(x_{10}) = 10/11 = 0.91$; for $n = 100$ the range of frequencies extends roughly from $F(x_1) = 0.01$ to $F(x_{100}) = 0.99$. In view of the fact that the frequency range of interest in reliability analysis is $P(x) \ll 10^{-3}$ or $R(x) > 0.999$, the fitting of observations $F(x_m)$ by $P(x)$ in the range $0.1 < P(x) < 0.9$ does not justify extrapolation of the fitted probability function into the significant reliability range.

Therefore, statistical inference is relevant in reliability analysis only when sample sizes are sufficiently large to permit a valid distinction between various possible probability functions within the frequency range enclosed by the sample size. The mean or median of a distribution may be accurately enough estimated on the basis of 3 to 5 specimens, the estimate of the standard deviation may require as many as 10 to 15. However for the determination of the distribution function itself by pure statistical inference even one hundred specimens are certainly not enough.

Thus for instance Weibull[2] has shown that to make a significant distinction between the logarithmic-normal distribution and the third asymptotic distribution of smallest values ("Weibull distribution"), both skew functions of apparent similarity, a sample size larger than 1000 would be required. On the other hand, when these two probability functions are alternatively fitted to samples of size $n = 10$ or $n = 100$, a range within which no distinction between them is possible, significantly different frequencies are obtained in the range significant for reliability analysis.

Statistical inference can therefore not be applied, unless the sample size is very large, a fact which distinguishes structural reliability theory from industrial statistics: in the latter the emphasis is on the central tendency of the probability function and the (relatively narrow) variation about it, in the former the main interest is in its form in the extreme ranges. In industrial statistics methods of inference are used to differentiate between statistical populations characterized by their means and variances. In structural reliability analysis where the shape of the distribution function is significant, sufficient-

ly large sample sizes for the use of methods of statistical inference arise in general only from load-observations and load-records. Observations of material characteristics such as strength-parameters or intervals between failures are generally limited in number, the more severely the larger and costlier the system or the structural element. Statistical inference as a basis of reliability analysis is therefore limited to load analysis and, possibly, to strength analysis of mass produced small elements that can be tested with sufficient replication under adequate control to produce sufficiently large, statistically homogeneous samples. On the other hand, the distribution and associated probability functions for the strength of large structural elements and complete mechanical systems must be selected by probabilistic-physical reasoning. It is on this basis alone on which extrapolation from a small number of test-results or observations into the reliability range can be justified.

II.   RELATION BETWEEN TESTS AND STRUCTURAL PERFORMANCE

Statistical variables expressing the "mechanical strength" of the material are relevant to reliability analysis only if they are relevant to the structural performance for which the reliability is to be established. For this purpose the results of most standard mechanical tests used either for purposes of quality control or for purposes of comparison of materials are useless. Qualitative correlation between laboratory specimen tests and material performance in the structure can, in general, be established only for deformation characteristics, such as elastic moduli, creep-rates, yield-limit and damping on the basis of continuum mechanical concepts. Strength characteristics depend to such an extent on geometry, absolute size, surface conditions and environment that in the analysis only the results of such tests can be used that have been specifically designed to reflect the relevant performance of the material in the structure under the critical condition of failure with which the reliability analysis is associated.

Failure in structures is the result either of the exceedance by a very rare load intensity of the initial resistance of the structure to deformation instability ("collapse") or to rapid fracture ("ultimate load failure"), or of the exceedance by a somewhat less rare load intensity of the "residual strength" of the structure, which is the resistance remaining at any time t as a result of progressive damage produced in the course of the service during time t either by a large number of service load cycles of relatively high frequency of occurrence ("fatigue failure"), by a sequence of sustained service load-temperature-combinations ("creep-fracture") or by a number of combined load-temperature cycles. Both rapid and progressive failures are preceeded by extensive redistribution of the internal forces in the structure; they can therefore not be reproduced in tests of simple specimens. Except for conditions of instability failure goverened by plastic collapse which depends on the yield limit, "materials testing" for reliability analysis therefore differs significantly from the conventional materials testing procedures. It is not a material parameter obtainable from small specimen tests, but the rate of propagation in structural members and parts of cracks from unavoidable structural defects which emerges as the most important "material" parameter by which the "damage tolerance" of a structure is determined.

"Damage tolerance" is the capability of a structure to operate after suffering a limited extent of critical damage; it is of particular significance in aircraft structures. Structural reli-

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ability testing to establish "damage tolerance" can thus not be
related to conventional materials testing, since it involves the
testing of full scale structures or of primary structural parts
with respect to such aspects of material performance that are not
duplicated in standard mechanical tests on serially reproducible
small or medium sized specimens. The most significant observation
in structural reliability testing is the expected time to the first
appearance of damage in the structure and the rate at which such
damage propagates to produce actual failure. Such observation re-
quires tests of the actual configuration of the structure under
relevant operating conditions or under suitably accelerated serv-
ice conditions; it is unobtainable by specimen tests.

In view of the very small feasible number of full-scale struc-
tural tests or tests of structural parts the principal problem of
structural reliability testing is the selection of a physically
relevant reliability function for extrapolation into the reliabil-
ity range from the small number of test results. These results
can only provide an estimate of a suitable measure of central
tendency (mean or median value) of the "variable" such as the ex-
pected "ultimate strength" for structures under conditions in
which progressive deterioration of their carrying capacity by re-
peated or sustained loads is not a significant design considera-
tion, or the expected life to critical damage or to failure for
all other conditions. However, a measure of central tendency alone
is of not much use in reliability analysis unless it can be supple-
mented by a well-based estimate of scatter in the form of the
variance or standard deviation as well as by a physically based
argument for the selection of a specific probability function for
extrapolation. Even this is not enough in the case of "ultimate
strength". The probability function must be truncated somewhere
below the mean or median so as to prevent that the reliability
analysis be governed by the spurious probabilities arising from
the consideration of structures of practically impossible low
strength failing under the low loads of highest frequencies of
occurrence. Such truncation must reflect the existence of a nor-
mal production control in the process of structural assembly which
automatically ensures the elimination of unreasonably low strength
values.

The performance of full-scale tests to determine the "ulti-
mate strength" or carrying capacity is already standard practice
in aircraft construction. The performance of full-scale fatigue
tests of gust-critical aircraft structures is gradually being
accepted as a necessity, since comparison of fatigue lives com-
puted on the basis of the linear damage accumulation theory with

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the test life[3] shows ratios consistently below unity

[Test life/computed life] < 1

with wide scatter about a central value of roughly 2/3. It should
be noted however that comparison of the life estimated on the ba-
sis of tests with actual service life[3] also consistently pro-
duces ratios below unity

[Life to service damage/life to test damage] < 1

with wide scatter around a central value of roughly 1/3. Combining
the two ratios it would appear that the computed fatigue life of
a full-scale gust-critical aircraft structure over-estimates the
operational life by a ratio of roughly 5:1, with some scatter.

In view of the results of numerous series of fatigue tests
under programmed and random load amplitudes on material specimens
and small assemblies of various types, performed to investigate
the validity of the rule of linear damage accumulation, the above
result does not seem unexpected, since the majority of the results
of specimen tests show sums of cycle ratio substantially below
unity, unless specific conditions of geometry or loading have been
created to introduce residual compressive stresses of sufficiently
high intensity so as not to be affected by the applied cyclic
stresses. It should be noted however that the correlation between
the type of service fatigue failure produced in structures and
that observed in specimen tests is rather vague. Thus, for in-
stance, many structural failures are the result of fretting, a
failure type that is quite uncommon in well-designed specimen
tests in which fretting failure in the grips is rare. Hence the
above agreement between the tendency of the results of specimen
fatigue tests and of the fatigue performance of full-scale struc-
tures with respect to the values computed by the linear damage
rule is somewhat unexpected and not too much reliance should be
placed on it, unless it is validated for the particular struc-
tural configuration by at least one full-scale fatigue test under
a representative load spectrum.

III. ESTIMATE OF FATIGUE LIFE (TIME TO FIRST FAILURE)

An estimate of the scatter characteristic of fatigue tests of specimens, assemblies and structural parts, based on the representation of test data by the logarithmic-normal distribution, has been obtained by pooling the results from various sources (Fig. 1). It appears that a value of the standard deviation $\delta = \sigma$ (log N) $= 0.15 - 0.20$ is representative of most results in the long life range ($N > 10^6$ cycles). The associated coefficient of variation based on the mean ($\bar{N}$) is obtained from the relation

$$\left[\frac{\sigma(N)}{\bar{N}} \right]^2 = \exp (2.3026 \ \delta)^2 - 1 \tag{2}$$

On the basis of the plausible argument that the structure which fails first out of a population of structures subject to the same mission spectra is the weakest structure in the population and the extension of this argument to the second weakest structure, the asymptotic distribution of smallest values[5] with a positive lower limit may be considered physically relevant as a fatigue reliability function provided only the first few failures in a large population are used in the estimate of its parameters. Replacing the number N of cycles to failure as the statistical variable by the time t to fatigue failure under the operational load spectrum, which is permissible if the interval Δt between load cycles is uniform so that $t = N\Delta t$, or if it is governed by a homogeneous Poisson process with Δt as the mean interval, this function has the form

$$R(t) = \exp \left[- \left(\frac{t - t_o}{v - t_o} \right)^\alpha \right] \tag{3}$$

where v denotes the "characteristic" fatigue life for which $R(v) = e^{-1}$, α is a scale factor which increases with decreasing scatter and t_o is a lower limit of the variate, the "minimum life".[4]

The mean frequency of the m-th failure given by Eq. (1) is now compared with the probability of failure $P(t_m)$ at time t_m according to Eq. (3)

$$P(t_m) = 1 - R(t_m) = \frac{m}{n+1} \tag{4}$$

and therefore

$$\exp \left[- \left(\frac{t_m - t_o}{v - t_o} \right)^\alpha \right] = 1 - \frac{m}{n+1} \tag{5}$$

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It follows that

$$t_m = t_o + (v - t_o) \left[- \ln \left(1 - \frac{m}{n+1}\right) \right]^{1/\alpha} \tag{6}$$

The time to the first failure is obtained for m = 1.

Disregarding, in first approximation, the lower limit t_o the reliability function has the form

$$R(t) = \exp \left[- (t/v)^{\alpha} \right] \tag{7}$$

The expected time to the first failure (expected shortest life) is obtained from Eq. (6) with $t_o = 0$.

$$t_1 = v \left[- \ln \left(1 - \frac{1}{n+1}\right) \right]^{1/\alpha} \tag{8}$$

Introducing the relation between α and the coefficient of variation $\sigma(\log t)$

$$\sigma(\log t) = \pi/(2.303\alpha\sqrt{6}) \tag{9}$$

which has been plotted in Fig. 2, and the relation between the mean $\bar{t}$ and the characteristic value v

$$\bar{t} = v \, \Gamma(1 + 1/\alpha) \tag{10}$$

Eq. (8) can be transformed into a relation between the ratio $(t_1/\bar{t})$ of the expected time to first failure to the expected (mean) time to failure and the standard deviation $\sigma(\log t)$. This relation is plotted in Fig. 3 for the population sizes n = 20, 50, 200 and 1000.

Using these diagrams it can be seen that in a fleet of moderate size, such as n = 50, the expected time to the first failure for $0.15 \leq \sigma(\log t) \leq 0.20$ is $0.35\bar{t} > t_1 > 0.25\bar{t}$. For a larger fleet of n = 1000 and the same range of scatter the expected time to the first failure $0.13\bar{t} > t_1 > 0.08\bar{t}$. Hence, depending on the fleet size, observed times to failure of between one-third and one-tenth of the mean time to failure cannot be considered unusual. In fact, in view of the scatter associated with the expected times to first failure, values smaller than the estimate of the expected time t_1 will frequently be observed. Since the time to first failure is an extremal phenomenon, it is to be expected that its distribution is an extremal distribution of smallest values, which is the condition of sta-

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bility characterizing such distributions.

Within the range of $0.15 \leq \sigma(\log t) \leq 0.20$ the expected times to the second failure are $0.46\bar{t} > t_2 > 0.34\bar{t}$ for $n = 50$ and $0.21\bar{t} > t_2 > 0.14\bar{t}$ for $n = 1000$. This implies mean intervals between the first and second failures of $0.11\bar{t} \geq (t_2 - t_1) \geq 0.09\bar{t}$ for $n = 50$ and $0.08\bar{t} \geq (t_2 - t_1) \geq 0.06\bar{t}$ for $n = 1000$.

If the expected time to the first failure t_1 is specified as a design criterion, the mean time to failure for which a fleet has to be designed becomes a function of the anticipated size of the fleet. Thus, for instance, for an anticipated scatter in the fatigue performance of the structure characterized by $\sigma(\log t) = 0.15$ the design mean time to failure is $\bar{t} \sim 3t_1$ for $n = 50$ and $\bar{t} \sim 7.5t_1$ for $n = 1000$. The associated expected intervals to the second failure are therefore roughly $0.3t_1$ and $0.53t_1$ respectively. It is obvious that because t_1 is a statistical design for an expected value of t_1 also implies a certain risk of failure which can be expressed by a reliability function.

For $n = 2$ (smallest sample size) the value of t_1 is fairly close to the mean $\bar{t}$ ($0.88\bar{t} > t_1 > 0.81\bar{t}$). The expected life to first failure in such sample size provides hardly more information concerning the expected life to first failure in an associated population than the mean itself.

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IV.   USES OF "TIME TO FIRST FAILURE" IN RELIABILITY ANALYSIS

The difference between sample size and fleet size must be considered in the planning and evaluation of so-called "lead tests" in which a very small number of units of the fleet are subjected to tests under an accelerated service spectrum starting simultaneously or in advance of the operation of the fleet. In order for the first-failure in the lead test to occur before the first failure in the fleet the relation must be satisfied

$$t_{1L} < t_1(n) + t_o \quad \text{or} \quad \bar{t}_L = \beta^{-1}(t_o + \gamma\bar{t}) \tag{11}$$

where $\bar{t}_L$ is the required mean time to failure in the lead group of size n_L, t_o the interval between the start of the lead tests and the start of the service operation, and the coefficients β and γ are functions of n obtained from $t_{1L} = \beta\bar{t}_L$ and $\bar{t}_1 = \gamma\bar{t}$. Thus for $\sigma(\log t) = 0.15$, $n_L = 2$, $n = 50$ and $t_o = 0$: $\bar{t}_L < 0.4\bar{t}$, a result which indicates that the necessary intensification or acceleration of the lead test load spectrum must be such as to reduce the mean time to failure by a factor of more than 2.5 if the lead test is to be of any use. For a large fleet ($n = 1000$) $\bar{t}_L < 0.15\bar{t}$ which implies a reduction of the mean time to failure is the lead test by a factor of 6.7.

A considerable advantage of the use of the time to first failure in reliability analysis arises from the fact that the precision of its estimate increases with n more rapidly than the precision of the estimate of the mean or of the characteristic value. Since the probability for the minimum of n observations from an initial extreme value distribution to exceed t_1 is [5]

$$R(t_1) = \exp\left[-n\left(\frac{t_1}{\tilde{t}_1}\right)^\alpha\right] \tag{12}$$

where $\tilde{t}_1$ is the expected value of t_1 and the scale factor α remains unchanged, it is obvious that the distribution of the minima contract with increasing number of extremes. The variances of the minimum of the smallest values

$$\left[\sigma(t_1)\right]^2 = \sigma^2 n^{-2/\alpha} \tag{13}$$

decrease with increasing n.

In another application of concepts of order statistics in reliability analysis the two shortest observed times to failure, being considered the weakest members of a sample, might, in first

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approximation, be considered to belong to an extremal distribution of unknown parameters. A rough estimate of the two parameters v and α of this distribution can be obtained by solving the two Eqs. (6) for $m = 1$ and $m = 2$ under the simplifying assumption $t_0 = 0$. The resulting distribution can be used to predict the expected times to first failure in larger samples.

Thus, for instance, the shortest times to fatigue damage observed in a sample of 40 aircraft[6] were $t_1 = 1500$ hrs and $t_2 = 1733$ hrs. Solving the two equations

$$t_1 = v \left[- \ell n \left(1 - \frac{1}{41} \right) \right]^{1/\alpha}$$

$$t_2 = v \left[- \ell n \left(1 - \frac{2}{42} \right) \right]^{1/\alpha}$$

the values for the parameters of the extremal distribution are $v \doteq 1.78\, t_2 = 3100$ hrs and $\alpha = 5.1$ or $\sigma(\log t) \doteq 0.11$; therefore $\bar{t} \sim 0.92 \times 3100 = 2850$. Hence, for a fleet of $n = 200$ the expected time to the first fatigue damage of the type observed would be $t_1 = 0.38\bar{t} = 1100$ hrs.

In view of the considerable difference between the mean or characteristic times to failure and the expected time to first failure, and of the effect of the parameter α on this difference, the fatigue-sensitivity factor[7] of the structure might be related to the expected times to first failure in a fleet of a certain size rather than to the risk of failure of mean times to failure in a fleet of indeterminate size. If v_U denotes the expected time to ultimate load failure associated with an exponential reliability function (with $\alpha = 1$), and v_F the expected time to fatigue failure with $\alpha = \alpha_F$, the fatigue sensitivity factor f based on expected times to failure[7]

$$f(t) = \frac{r_F}{r_U} = \frac{\alpha_F}{v_F} \left(\frac{t}{v_F} \right)^{\alpha_F - 1} v_U \qquad (14)$$

while an alternative definition of the fatigue sensitivity factor of the form

$$f = \frac{t_{1U}}{t_{1F}} = \frac{v_U \left[- \ell n \left(1 - \frac{1}{n+1} \right) \right]}{v_F \left[- \ell n \left(1 - \frac{1}{n+1} \right) \right]^{1/\alpha_F}} = \frac{v_U}{v_F} \left[- \ell n \left(1 - \frac{1}{n+1} \right) \right]^{(\alpha_F - 1)/\alpha_F}$$

$$(15)$$

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relates the fatigue sensitivity at time t_1 to fleet size. It is obvious that because of the short times t_1 to first failure the constant fatigue sensitivity at t_1 according to Eq. (14) is much lower than that defined by Eq. (13) for $t > t_1$ which is an increasing function of time. This is mainly due to the fact that the expected time to the first failure for the exponential distribution ($\alpha = 1$) characteristic of ultimate load failures is a much smaller fraction of the expected time to failure than for the extremal distributions ($\alpha_F > 1$) characteristic for fatigue failures. Thus, for instance, for n = 50 the expected time to first failure for an exponential reliability function is $t_1 = 0.0202\ v_U$, while for n = 200 this time is $t_1 = 0.005\ v_U$. Therefore very long mean times to failure that are governed by chance do not provide adequate safety against premature failures even in medium size populations.

Comparing the above expected times to first ultimate load failure with those computed for fatigue failures it appears that in order to ensure equal expected times to first failure for ultimate load and fatigue failure (disregarding the fact that for long operational periods ultimate load failures become failures of the fatigue-damaged structure, because of the relatively short times considered) the ratios between v_U and v_F required to prevent premature chance failures are $(v_U/v_F) > 10$ already for n = 50 and much higher for larger fleets.

The above analysis illustrates the ambiguities encountered in the comparison between life estimates based on tests and fatigue lives observed in service, and the necessity of comparing not mean lives but expected lives to first failure in the reliability analysis of even a moderately large population based on the results of a very small number of tests. Comparison of means alone are quite misleading in the reliability assessment of such a population particularly when, as in the case of large structures, the purpose of the reliability analysis is the prevention of failure of even a single member of the population. Because this requirement seems to be the only rational requirement for the design of structures failure of which is in effect, inadmissible, such as large transport aircraft, it appears that design for a specified time to first failure associated with a reasonably low risk should replace the current approach of design for a specified mean service time coupled with a vague "scatter factor".

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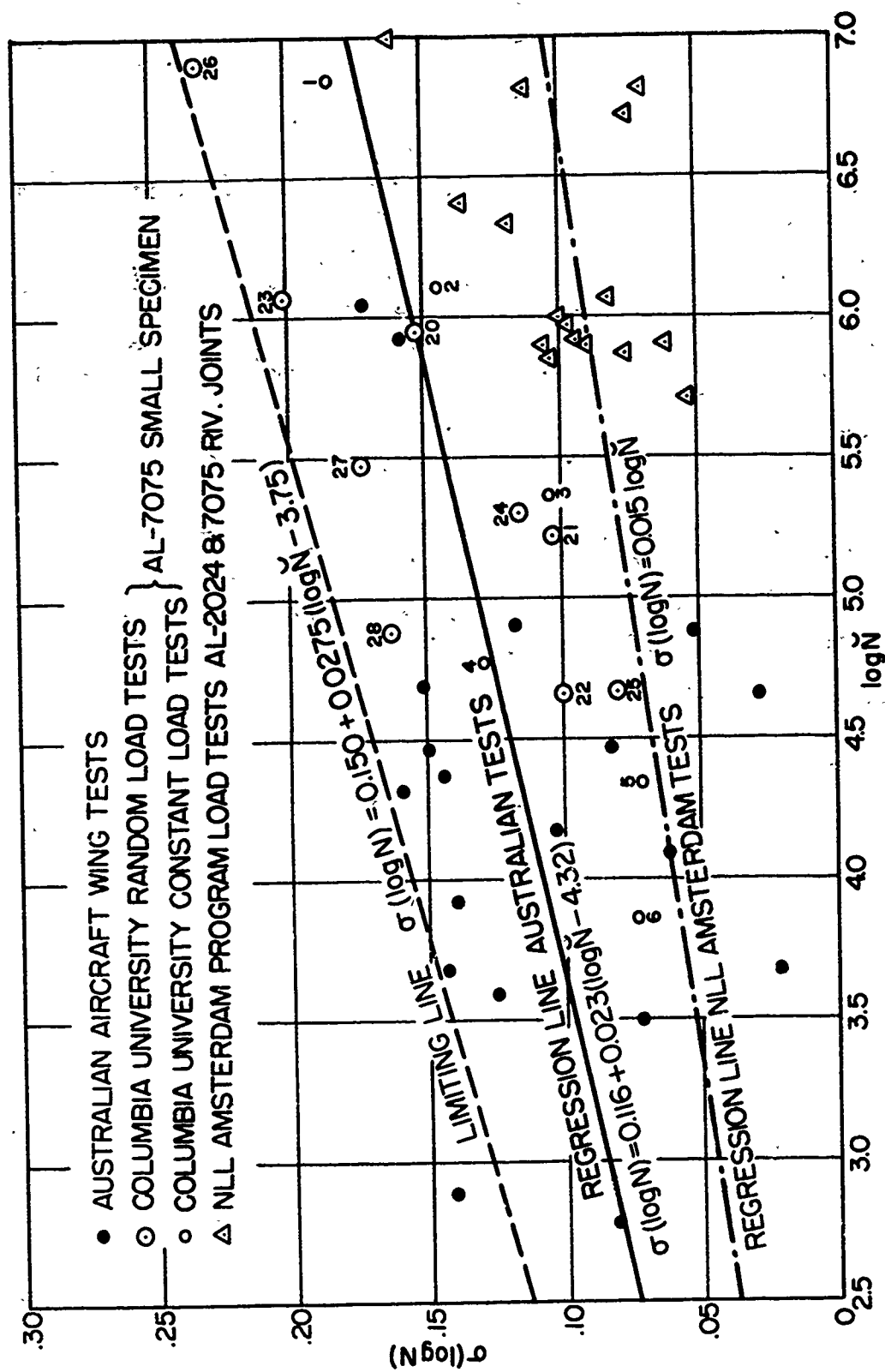7. Freudenthal, A. M., WADD Tech. Rep. 61-53, July 1961.

Fig. 1   Relations between $\delta = \sigma$ ($\log_{10}N$) and $\log \check{N}$ obtained in various constant amplitude, program and random fatigue tests. (from WADD Tech. Rep. 61-53, 1961)
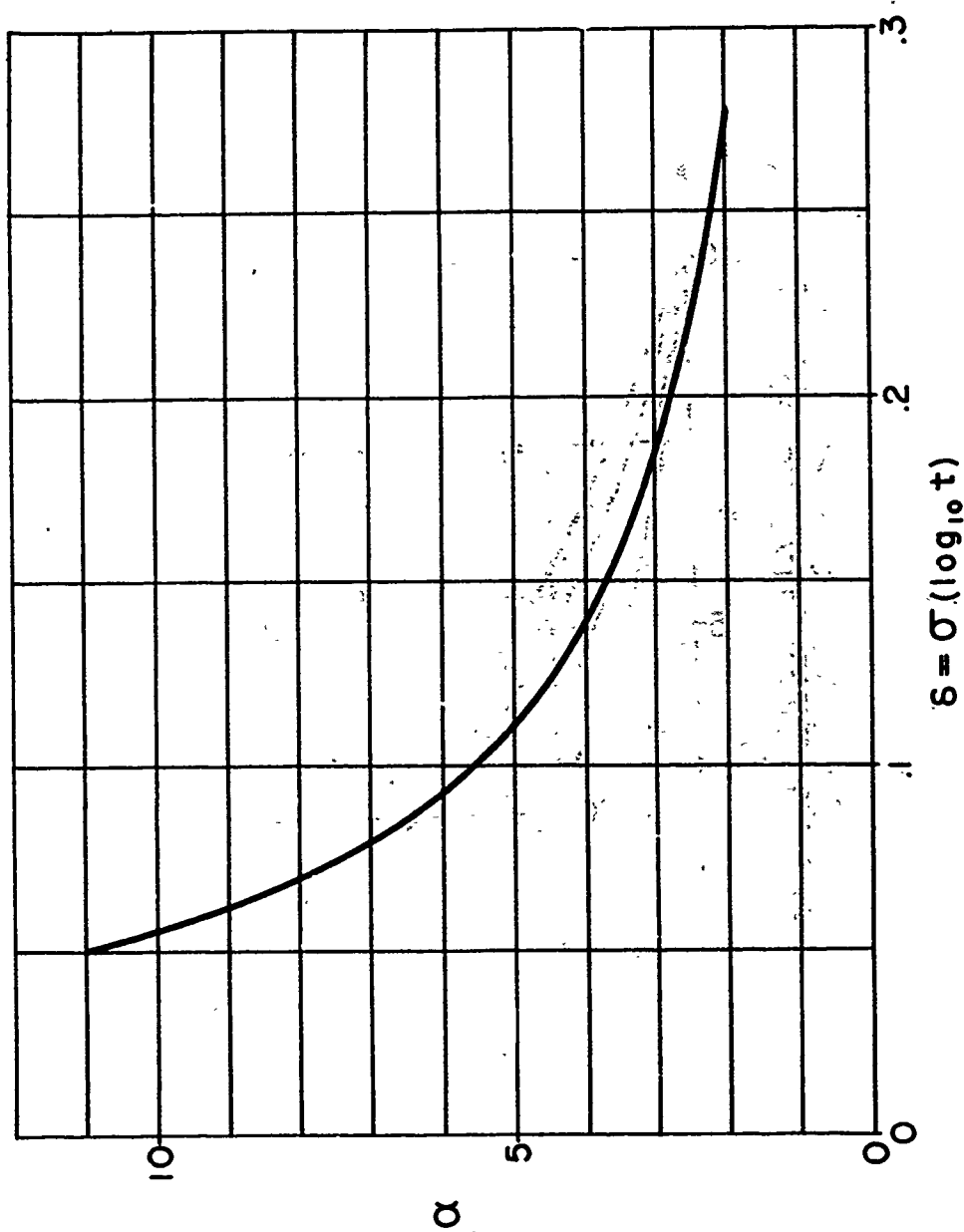
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Fig. 2 Relation between $\delta = \sigma(\log_{10} t)$ and the scale parameter α of the Third Asymptotic Distribution of Smallest Values.
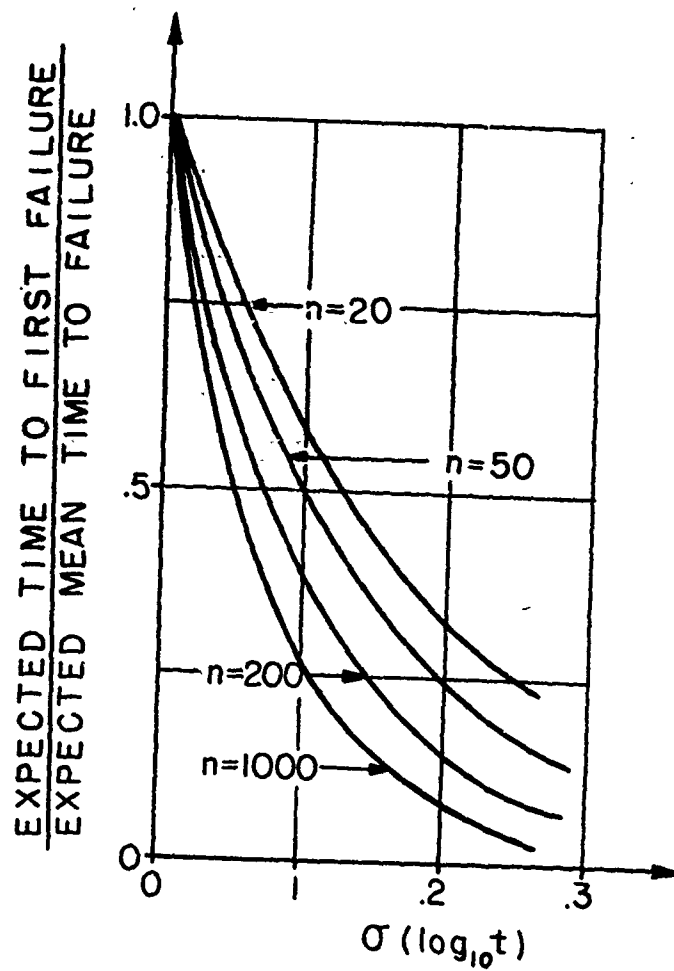
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Fig. 3   Computed ratios of expected time to first failure to expected mean time to failure as functions of $\delta = \sigma (\log_{10}t)$.

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13. ABSTRACT

A new approach to structural reliability analysis based on order statistics is introduced by considering the expected time to the first failure in a fleet of specified magnitude. Because in the design of large structural units, such as transport aircraft, failure of even a single unit must be prevented, reliability analysis and design for a "mean time to failure" seems to be an unjustified extension of the use of methods of reliability analysis developed for inexpensive mass-produced items of relatively short service lives to the reliability assessment of expensive, large units. A method for the estimate of the expected time to the first failure is outlined and the implications of the use of this time in reliability analysis and design are discussed.

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14. KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
Reliability						
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